

AUTOMATED HARMONIC ANALYSIS ON COMMON PRACTICE MUSIC

by

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ABSTRACT

The following thesis reports on work performed to replicate and improve upon an algorithm for the Roman numeral harmonic analysis of homophonic choral music. Improvements made aimed to expand the reach of the algorithm, which was initially implemented specifically with Bach chorales in mind, to the broader period of common practice art music and the homophonic choral music within it. The thesis concludes by exploring potential Web-based applications for the algorithm's new implementation, with particular focus on the area of music theory education. The associated code has been included as a supplement to this paper on the NC Docks archive.

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I. INTRODUCTION

In music history, the common practice era defines a period roughly spanning the mid-to-late Baroque, Classical, and early-to-mid Romantic periods of art music (c. 1650-1900). Music of the common practice era is distinguished by a tonal system where melody and harmony are defined by their relationship to a major or minor tonal center. Music theory describes these harmonic relationships with Roman numeral notation — where ‘*I*’ indicates the tonic major chord, ‘*V*’ signifies the dominant chord to the tonal center, as so on. These relationships can be notated more granularly with the addition of figured bass, which conveys inversion information — ‘*V*⁶’, for example, indicates a first-inversion dominant chord, where the ‘6’ signifies the root of the chord is a sixth above the bass. Figure 1.1 shows an example tonal center, the key of C major and C minor, with its accompanying Roman numeral notations; Fig. 1.2 shows examples of figured bass indicating inversion information.



Figure 1.1: The chords in the tonal center of C/Cm and the Roman numeral notations for their harmonic relationships to that center



Figure 1.2: figured bass notations for inversions of three notes chords, or ‘triads’ (first measure), and four note chords, or ‘tetrads’ (second measure)

With this notation, one can describe the harmonic content in a common practice work with a series of figured Roman numerals, indicating the bass line and each chord's relation to a tonal center, a key part of a music theory student's education. Of all the subgenres of music within the common practice era, homophonic choral music remains the most useful style for music theory pedagogy, in part due to its essential prevalence throughout the common practice era as a whole, as well as to its relative simplicity compared with its polyphonic cousins. In homophonic choral music, all voices move in largely the same rhythm, usually set to a text — outside of the classroom and concert hall, this homophonic style of music prevails primarily in religious hymnals.

With the rise of computers in the past several decades, leveraging computational automation to aid in primary, secondary, and tertiary education has become an increasingly popular practice, and music theory should be no exception. Indeed, as early as 1999, Professor Heinrich Taube of the University of Illinois Urbana-Champaign expressed interest in automating the grading of student music theory assignments. As part of his efforts to fulfill that goal, he created an algorithm for the automated harmonic analysis of homophonic choral music, which he detailed in [1] in a paper submitted to the *Computer Music Journal*.

The purpose of this paper is to report on my own efforts over the past semester to recreate and improve upon Taube's algorithm using modern technologies. Through my efforts, I have openly addressed some of the points of future work Taube listed in his paper and also went further into making the algorithm applicable to as much of common practice choral music as possible. In particular, I have implemented updates to certain areas of the algorithm to bring the analysis to a higher level which closer represents how humans interpret music (scores are

provided in the appendix for the reader to judge on this point). Concluding the paper, I will ruminare on the additional applications of the algorithm made possible by the technologies I used in its implementation.

II. TAUBE'S ALGORITHM

The algorithm as outlined by Taube in [1] takes a music engraving file for a piece of music as input and outputs an engraving file containing the Roman numeral analysis for the piece. The engraving file contains a plaintext representation of all the notes present in the piece, as well as additional data pertaining to how to print those notes to a page which the algorithm ignores. The Roman numeral analysis is formatted with each unique tonal center receiving its own line containing the relevant numerals. For the duration of this section, I will be referring to Fig. 2.1 below as a simple example on which to apply the algorithm.



Figure 2.1: A very simple musical example, used to illustrate the algorithm throughout this section

The algorithm begins by parsing the input score into a series of vertical sonorities - henceforth referred to as 'verticals' in this paper - containing all the notes sounding at a

particular point in the piece. The verticals are represented with a chromatic vector of its notes, which means that octave displacements of the same note appear as only one note in the vertical representation. For example, the first vertical in the line in Fig. 2.1 would be represented as containing the notes C, E, and G. Every note articulation in the score will denote the start of a vertical, so the example above will be represented as a series of seven verticals. Table 2.1 below shows a summary of these verticals: at this point in the process, all columns in the table but the ‘classification’ column have been determined.

Table 2.1: encoding and classification of verticals in Fig. 2.1

Vertical #	Notes	Vector	Duration (beats)	Bass	Classification
1	C, E, G	100010010000	0.5	C	Cmaj
2	C, D, E, G	101010010000	0.5	D	Cmaj, D NCT
3	C, E, G	100010010000	1.0	E	Cmaj, first inversion
4	C, F, A	100001000100	1.0	F	Fmaj
5	D, G, B	001000010001	0.5	G	Gmaj
6	F, G, B	000001010001	0.5	G	G7 (partial), F functional dissonance
7	C, E, G	100010010000	4.0	C	Cmaj

These verticals then go through the process of classification as a series of chords and non-chord tones (NCTs)¹. No functionality is yet assigned to the verticals, merely a basic classification of the chord itself. In the Fig. 2.1 example, the first vertical will be interpreted as a root-position C major chord, and the second vertical a C major chord with a D as an NCT. Partial chords are also identified in this step - e.g. the penultimate vertical in the above example will be

¹ In his paper, Taube refers to NCTs as non-harmonic tones (NHTs). The paper will be using the term NCT instead.

recognized as a partial G dominant 7th chord (missing its fifth, a D). In this step, verticals are also classified by whether or not they have a functional dissonance, such as a dominant 7th implying a tonic a fourth below the chordal 7th or a diminished chord implying a tonic a half step above the chord's root. In the provided example, the penultimate G7 vertical contains the functional dissonance F, which implies a tonal center of C - indeed, the final vertical confirms that implication. Table 2.1 shows all the classification data determined for the Fig. 2.1 example in the final column.

At this point, as Taube describes in [1], every single articulation has been classified as a harmonic change. This behavior unfortunately yields the 'over-interpretation' of what may be better explained as melodic embellishment around or within a chord. For example, the bass line's movement in the first beat of Fig. 2.1, currently classified as two verticals (1 and 2), should instead be explained as the bass melodically passing between two chords. Similarly, the alto line's movement in the fourth beat (verticals 5 and 6) should be classified as melodic movement within the beat's G7 chord instead of as two separate chords. To address this, Taube proposes a melodic consolidation process which would integrate the analysis so far into a higher level harmonic structure that allows for individual articulations to be accounted for as melodic movement where necessary. As of the writing of [1], however, Taube lets this process go unimplemented, instead leaving it for his proprietary future work. In section V, I will describe the consolidation process used in my second implementation of Taube's algorithm.

Once all chords and NCTs have been identified, the verticals then go through a process of tonal center identification. To determine the locations of tonal centers, the algorithm analyses the set of verticals for three categories of occurrences:

- Cadence points: As ‘resting points’ in a composition, cadence points play an integral role in determining an underlying harmonic structure. In the algorithm as implemented by Taube, this step is limited to finding fermatas amongst the verticals, as he limited his testing of his algorithm to Bach chorales, which mark all cadences with a fermata; in section VI, I will describe a more generalized cadence detection scheme in my second implementation of the algorithm.
- Touches: A ‘touch’ occurs when a chord is preceded by either a major chord with a root a perfect fifth above the touched chord’s root, indicating a possible $V \rightarrow [I | i]$, or a diminished chord with a root a minor second below, indicating a possible $vii^\circ \rightarrow [I | i]$.
- Implications: An ‘implication’ occurs when a chord contains a functional dissonance, such as a dominant seventh, indicating a possible V^7 which implies a tonal center a perfect fifth below the chord’s root, or a diminished interval, indicating a possible vii° or $vii^\circ7$ which implies a tonal center a minor second above the chord’s root.

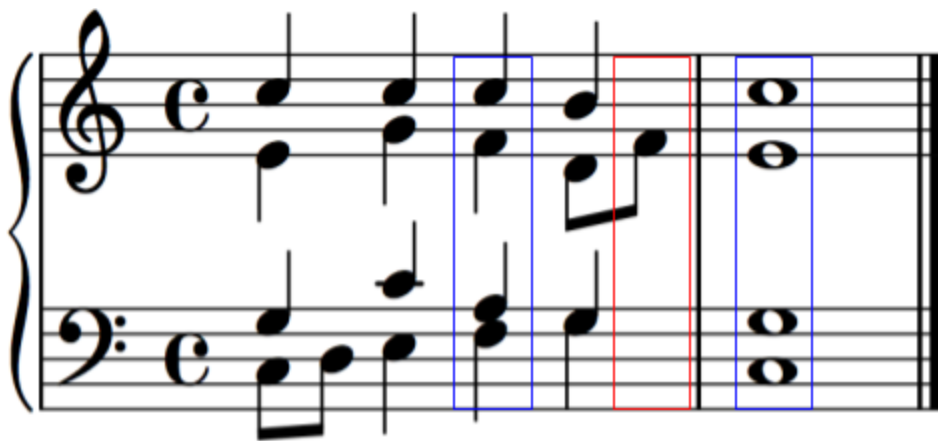


Figure 2.2: The same musical example as Fig. 2.1, with touches indicated in blue, implications in red; with more robust cadence detection, the final chord would be marked as a cadence as well

With the above items identified, tonal center confirmation then occurs with any of the following cases:

- A tonal center being implied and then touched, indicating either a possible $V^7 \rightarrow [I | i]$ or a $[vii^\circ | vii^{\circ 7}] \rightarrow [I | i]$
- A touched cadence, i.e. $V \rightarrow [I | i]$ where the cadence point occurs on the $[I | i]$
- A touched successor to a cadence, i.e. $V \rightarrow [I | i]$ where the cadence occurs on the V

In the example for Fig. 2.1, only one tonal center is confirmed at the final vertical of the piece, which is the result of a touched implication (the G7 implies C major).

With tonal centers identified, the algorithm then proceeds into the functional analysis stage. The process involves taking in every tonal center confirmation and ‘filling in’ between the confirmed centers to create a map of the piece’s functional structure. To do this, for each consecutive pair of tonal centers, the algorithm starts with the left center and determines how far right into the series of verticals the chords can be explained as part of that tonal center’s harmonic model, then does the same with the right center in the opposite direction. With these two ‘areas of sway’ identified for the centers, one of three situations arises:

- The regions cross to make a shared region: in this case, a pivot chord must be determined. For his implementation, Taube simply selects the last chord in the region, which he admits to be a suboptimal scheme; section III of this paper includes a description for a change I made to pivot selection.
- The regions adjoin, i.e. one starts immediately after the other ends: the algorithm need do nothing in this case, since all verticals are singly accounted for.
- The regions fail to adjoin and have a gap in between them: the algorithm in this case goes through a process Taube calls the ‘modulation model’, which attempts to resolve the first unexplained chord as a modulation into a new, previously unconfirmed tonal center, using

known possibilities for tonal center shifts (e.g. *V/vi* or *iii/III*, among others); the new tonal center is then added to the set of confirmed tonal centers.



Figure 2.3: a simple musical example illustrating functional analysis - tonal center confirmations are highlighted in purple, areas of sway for the second pair represented with arrows

Fig. 2.3 above is provided to illustrate the tonal center pairing process since the 2.1 example only has one tonal center confirmation. With the second pair of tonal centers (the second and third center confirmations) selected, the process determines the second beat of the third measure to be the final chord in the region that can be explained as part of the tonal center of D (as shown with the bottom arrow), and the third beat of the second measure to be the first chord in the region that can be explained as part of the tonal center of G (as shown with the top arrow). Since the regions overlap, the algorithm then determines a pivot chord for the region, a process I will describe in section III with the same example.

III. FIRST IMPLEMENTATION

For my implementation of Taube's algorithm, I chose NodeJS as the programming language and Lilypond as the music engraving language. I chose these languages because they are both open source and therefore make further extension upon the resulting program in the future a more reliable possibility. I chose NodeJS in particular to prepare the program for eventual interactions with other Web technologies, which I will explain further in section VIII. I also designed the program to make it easier to use a different engraving language should the need

arise, as is also explained in section VIII; Lilypond merely serves as the first engraving language for testing this algorithm.

In order to simplify different steps of the algorithm, the program uses a series of hash tables which represent chords and their possible functional explanations in a single location. This technique allowed me to combine the tonal center and modulation models into a unified harmonic model, a goal Taube listed as part of his future work in [1]. A chord is identified in one of the hash tables first by a bit-string vector denoting the chromatic notes in the chord (e.g. a C major triad is represented by “100010010000”) then further by the exact spelling of the chord, allowing for enharmonic equivalents of the same chromatic vector to be interpreted differently. For example, [C, E, G, B-flat] and [C, E, G, A#] are enharmonically equivalent, but serve very different functions - the former serves as a dominant seventh chord in the tonal center of F, while the latter serves as a German augmented sixth chord in the tonal center of E. In the hash tables, each chord also stores the type of functional dissonance the chord has (if any), the root of the chord, whether the chord is major, and data pertaining to how the chord functions in different keys, including possible tonal centers for when the chord becomes unexplained in the functional analysis. For example, a C major triad can serve as *I* in the key of C, *IV* in the key of G, *III* in the key of A, and so on, and when found to be unexplained most likely indicates a modulation to the key of F (as a $V \rightarrow I$).

In total, three hash tables are used for identifying chords: one for ‘straight’ three- or four-note chords (also known as ‘triads’ and ‘tetrads’, respectively), one for basic alterations to those straight chords (i.e. suspended fourth chords), and one for partial spellings of those straight chords (e.g. no-five spellings). For the chord classification step, the program first checks to see if

the chord's vector and spelling is in the standard chords table, then failing that checks the altered chords table. If no match is found in either table, it then checks to see if the vertical can be explained as a chord containing non-chord tones (NCTs) by masking the vertical's vector with vectors in first the standard chords table then the altered chords table and seeing if the masked vector matches the table's vector, then classifying all other notes in the vertical as NCTs in the case of a match. If the masking checks still yield no results, the program then checks to see if the vertical can be explained as a partial chord, then as a partial chord with NCTs if no matches still. If at the end of the process no match has been found, the vertical is labelled as a non-chord ('NC') which will subsequently be skipped over during functional analysis.



Figure 3.1: a simple musical example illustrating how better pivot selection helps considerably - a pair of tonal center confirmations are highlighted in purple, with arrows representing their regions of sway

In the first implementation of Taube's algorithm, I remained true to the description of his first implementation, with a couple of small changes. First, I added the functionality of declaring a vertical before a full rest (i.e. a moment where no voices have any notes) a cadence point to the cadence detection scheme, a first step in broadening the algorithm's reach to any homophonic choral music in the common practice era. I also altered the pivot selection scheme in the functional analysis to search for any cadence points in a shared region: if any cadence points exist, then the tonal centers are made to adjoin at the first cadence in the region, with the cadence being the ending point for the first tonal center in the pair. This change prevents the program

from declaring modulations to occur in the middle of a phrase when the first part of the phrase is better explained as functioning within the second tonal center. In the Fig. 3.1 example above, the old pivot selection process would extend the D tonal center confirmed at the first cadence all the way to the second beat of the following measure (as shown with the bottom arrow), whereas the updated pivot selection would correctly classify the chords after the cadence as part of the G tonal center confirmed at the final cadence.

IV. FIRST IMPLEMENTATION RESULTS

For testing my implementations of Taube’s algorithm, I took Lilypond scores from the Mutopia Project scores repository — which posts scores under Public Domain and Creative Commons licenses — and ran each implementation on those scores. Upon running the first implementation of the algorithm, we have results that are technically correct, but suffer from a couple of problems. Figures 4.1 and 4.2 both provide examples to illustrate these problems.

C: - - - - -
 G: I I IV⁶ vi V⁶ I iii⁶ V vi IV I⁷₅ IV⁴₂ vii^{°6} I V

Figure 4.1: The first line of BWV 269, analysed with the first implementation of the algorithm - analytical anomalies highlighted in purple

Of the two problems, excess granularity proves to be the most prevalent. As explained in section II, excess granularity in the analysis results in voice movement which would otherwise

be classified as melodic motion instead being classified as harmonic changes. As shown in the Fig. 4.1 example, three instances of excess granularity are present in the first phrase alone: the first two instances should be explained as melodic motion within a seventh chord ($IV \frac{6}{5}$ and $iii \frac{6}{5}$, respectively), and the third instance should be explained as neighbor-tone movement within the IV chord. Similarly, Fig. 4.2 below shows two instances of melodic motion within a chord being marked as two different chords. Figure 4.2 also shows an example of the second problem of poor pivot selection: since robust cadence selection has not been implemented yet and the cadence in measure 4 has gone undetected, the four chords after the cadence have been analysed as part of the tonal center of D from the previous phrase, when they should be analysed as part of the tonal center of G in the following phrase. This problem does not occur in the Bach chorales, since all cadences are accounted for by fermatas.

The image shows a musical score for Haydn's "Deutschlandlied". It consists of two staves. The first staff is in the key of D major (indicated by two sharps) and contains the lyrics: "al - les in der Wein und deut - scher deut - sche Va - ter -". The second staff is in the key of G major (indicated by one sharp) and contains the lyrics: "Welt, wenn es stets zu Schutz und Sang sol - len in der Welt be - land! Da - nach lasst uns al - le". Below the staves, there are chord annotations. The first staff has annotations: $V \frac{3}{4}$, $V \frac{6}{5}$, I, ii^6 , vii^{*7} , vii° . The second staff has annotations: I, IV, I, IV, I, V, $V \frac{6}{5}$, I. A purple box highlights the IV, I, IV, I sequence in the second staff, and another purple box highlights the vii^{*7} , vii° sequence in the first staff.

Figure 4.2: a (cross-staff) three measure excerpt from Haydn's "Deutschlandlied", illustrating both excess granularity and suboptimal pivot selection - first line represents the key of D, second the key of G

To put an objective score on the accuracy of the implementations' results, I devised a scoring system which places a number on the results for a piece on either implementation. To do

this, I created ground truth files for the analysis results I expect for each piece, then used the Unix `wdiff` command to compare the results of an implementation run on a piece with the expected results for that piece. The number of differences produced by the `wdiff` command is then halved — accounting for one change in an analysis requiring two changes to a Lilypond file — and negated, producing a scoring system in which a score of 0 denotes a perfect score. Tables 4.1 and 4.2 below show the scores for all the pieces tested on the first implementation.

Table 4.1: Scores for first implementation on Bach chorales

BWV number	259	264	269	277	347
Score	-39	-12	-16	-30	-19

Table 4.2: Scores for first implementation on other common practice works

Piece	Score
Haydn - <i>Deutschlandlied</i>	-20
Haydn - <i>Tantum Ergo Sacramentum</i>	-6
Naujalis - <i>Vexilla Regis Prodeunt</i>	-6

As the scores show, analysing Bach chorales with the first implementation yields many cases of excess granularity, with BWV 259 exhibiting the most instances. Compositions from the Classical era seem to suffer not nearly as much from this problem, which is a testament to the more fundamentally homogeneous harmonic texture of Classical music, as compared with Baroque music with its more contrapuntal and polyphonic influences. Haydn’s “Deutschlandlied” is an exception to this trend, foreshadowing the resurgence of more elaborate musical textures that would occur in the Romantic era. With these scores on hand, we can now begin to explore improving upon the algorithm to bring the analysis closer to the higher level expected from the harmonic analysis of common practice music.

V. CONSOLIDATION ANALYSIS

Further improving upon Taube's algorithm must necessarily start with implementing a harmonic consolidation process, which goes unimplemented by Taube [1]. As explained in section II, this consolidation process will account for any 'over-interpretation' of melodic motion as harmonic motion and subsume that melodic motion into a higher-level harmonic structure interpretation. This generalization of the harmonic structure of a piece will yield a more 'natural' final analysis by bringing the granularity of the analysis closer to how a human would interpret the piece.

With this idea of smoothing over excess granularity in mind, I started my implementation of the consolidation process by making the program initially suspicious of any supposed harmonic movement within a beat. This suspicion does not act immediately as a verdict, however, as harmonic rhythm may not coincide with the metrical beat of the piece. Instead, the offending verticals are analysed together for a match to a set of consolidation rules which determine whether and how to merge those verticals into a single harmonic point. The rules have been written as generally as possible in order for every rule not to apply to only one case but to be applicable across common practice music. One of two actions can occur for each rule:

- Merge and reinterpret: the verticals are merged directly into one vertical and reinterpreted as one chord — e.g. the verticals [C E G] and [E G B] would yield a Cmaj7 vertical.
- Give precedence to a vertical: the verticals are merged by labelling any notes present in the beat but not in the vertical with precedence as non-chord tones (NCTs) — e.g. the verticals [C E G] and [C D G] with precedence on the first would yield a C major vertical with D as an NCT.

ii	V ₁	ii	vii ^{o6}	vii ^{o6}	I	-	-	-	-	-	-	-		
-	-	-	-	-	-	V	I	IV ^o	IV	I ₁	V ⁴	V	V ⁷	I

Figure 5.1: the final line of BWV 264, as interpreted in the keys of D and G by the first implementation - beat one of the first measure shown qualifies for a melodic motion rule in the consolidation process

Most of the consolidation rules fall under one of two categories of consolidation: general harmonic stasis and excess melodic movement. Figure 5.2 below illustrates two examples of general harmonic stasis: if the roots of the interpreted chords within a beat are all the same, then they can be explained as a single merged chord for the beat, regardless of the motion within the beat on that chord. The first measure of the Fig. 5.1 example above provides a sample excess melodic movement example: the A in the middle of the first beat in the tenor line is best classified as a neighbor tone for the B on each side of it, which functions as the fifth of an E minor chord. A larger number of rules fall under the melodic movement category, and these rules are structured around how many voices have moved and the harmonic relationships between the possibly over-interpreted chords. In the case of the first beat in Fig. 5.1, the program recognizes the first vertical in the beat and the first vertical in the next beat to be identical and that only one voice has moved in between those verticals, and so it gives precedence to the first vertical.

Fig. 5.2: the final measure of BWV 259, as interpreted in the key of E by the first implementation - beats one and two each fall under the same harmonic stagnancy consolidation rule

Figure 5.3: A partial-measure excerpt from BWV 347, as interpreted in the key of A by the first implementation - first beat is caught by the no-five consolidation rule, second beat illustrates a special consolidation case

Additional consolidation rules exist to capture some special cases as well, including those illustrated in Fig. 5.3 above. As a design decision, I did not include no-three chords in the partial chords hash table, and because of that no-three verticals are labelled as non-chords. These verticals are accounted for in the consolidation process by merging them with their inevitable full-chord companion within the same beat, the first beat of the Fig. 5.3 excerpt being a prime example (indicated with a red box). Additionally, the first implementation suffers from an inability to ascertain pedal tones, a problem Taube himself recognized [1]. Because of this, the second half of beat two in the Fig. 5.3 excerpt (indicated with a blue box) is interpreted as a $D_{sus\#4}$ within the tonal center of A, when in reality it should be recognized as a partial $G\#dim$ with an A pedal NCT. The second implementation's consolidation process provides a rule for these cases as well.

VI. CADENCE POINT IDENTIFICATION

With the higher-level harmonic movement structure now identified, more robust cadence point identification presents itself as the next logical step in improving the algorithm. Cadence detection acts as a pivotal part in both tonal center identification and, in my implementation, pivot selection. As explained in section II, cadence points represent ‘resting points’ in a composition and therefore represent crucial points in the tonal structure of that piece.

The image shows a musical score excerpt with two staves (treble and bass clef) and seven lines of Latin lyrics. A blue box highlights measure 8, which contains a cadence. The lyrics are: 1. ste - ri - um, qua vi - ta mor - tem; 2. lan - ce - æ mu - cro - ne di - ro; 3. car - mi - ne, di - cen - do na - ti -; 4. pur - pu - ra, e - lec - ta di - gno; 5. sæ - cu - li, sta - te - ra fa - cta; 6. tem - po - re: pi - is a - dau - ge; 7. spi - ri - tus: qui - bus cru - cis vi - . Below the staves, Roman numerals indicate the chord progression: - - iv V - i - i - i VI° III° - - - - - vii° ii - - - - - I° iii vi.

Figure 6.1: an excerpt from Naujalis's "Vexilla Regis Prodeunt", as interpreted in the keys of E and G by the first implementation - measure 8 shows an as-yet-undetected cadence

The cadence point identification process in my second implementation of the algorithm follows a very simple paradigm. Given cadence points are ‘resting points’ in a piece, they can essentially be identified by finding a pause in the harmonic rhythm — and a pause in harmonic motion can be identified by the length of a vertical being sufficiently long in relation to all the other verticals. The process therefore consists of selecting the longest verticals in the piece, first by selecting all verticals two beats or longer in length (where the length of a beat is determined from the piece’s meter) then by removing verticals two beats in length from the selected set if longer verticals exist. Finally, if the piece contains a plagal cadence, the third to last and second

to last verticals are also removed from the selected set to ensure that the subdominant is not labelled as a tonal center confirmation.

This elegantly simple process proves surprisingly effective at finding cadences, thanks in part to the consolidation process defining a simpler harmonic motion timeline. In the example of Haydn's *Deutschlandlied* as shown in part in Fig. 4.2 in section IV, the first vertical in measure 4 is correctly marked as a cadence, since no motion slower than two beats long occurs. In the excerpt from Naujalis's *Vexilla Regis Prodeunt* shown in Fig. 6.1 above, which has a time signature of 3/2, the process will recognize the three-beat-long vertical in measure 8 as a cadence, and because of that detected cadence the algorithm will pull the subsequent pivot from E to G back to that cadence, as it should.

VII. SECOND IMPLEMENTATION RESULTS

C: - - - - -
 G: I I IV⁶ V⁶ I iii⁶ vi IV IV¹/₂ vii^{o6} I V

Figure 7.1: The first line of BWV 269 as analysed by the second implementation of Taube's algorithm - consolidation successes highlighted in purple

The first measure of Fig. 7.2 below shows examples of the success for the relatively simple harmonic stasis rules in the consolidation process: cases of a chord root having sway over an entire beat as shown in Fig. 4.2 in section IV have been correctly identified and merged into a single analysed vertical. Similarly, Fig. 7.1 above shows cases of excess melodic movement rules

in the consolidation process having similar success: instances of one or two voices moving within an established chord as shown in Fig. 4.1 in section IV have been correctly identified and merged or given precedence as desired. Additionally, the consolidation process proves not to be too sensitive, as shown in the second beat of the third measure in Fig. 7.1: the IV_2^4 and vii^{o6} were correctly left intact as they signify actual harmonic movement toward the beat three tonic.

Figure 7.2: The same (cross-staff) three measure excerpt from Haydn's "Deutschlandlied" as Fig. 4.2, this time analysed with the second implementation - first line denotes key of D, second key of G (to see approaching phrase, refer to the appendix)

The cadence detection process yielded success as well, as shown in the second half of Fig. 7.2: the cadence on the first beat of measure 4 has been identified and yielded the desired changes in the surrounding tonal centers' areas of sway marked for improvement in Fig. 4.2 in section IV. This change in interpretation successfully arose due to the effect of cadence detection on both tonal center confirmation and pivot selection. Because the cadence was detected, the touch immediately after the cadence was marked as a tonal center confirmation, and the detection of the cadence also directed the program to adjoin the tonal centers' areas of sway rather than place a common pivot chord in between them, yielding a more natural interpretation of the excerpt in Fig. 7.2.

Table 7.1: Scores for second implementation on Bach chorales

BWV	259	264	269	277	347
Score V1	-39	-12	-16	-30	-19
Score V2	0	0	0	0	0

Table 7.2: Scores for second implementation on other common practice works

Piece	Score on first implementation	Score after consolidation	Score after cadence detection
Haydn - <i>Deutschlandlied</i>	-20	-13	0
Haydn - <i>Tantum Ergo Sacramentum</i>	-6	0	0
Naujalis - <i>Vexilla Regis Prodeunt</i>	-6	-5	0

Tables 7.1 and 7.2 above show the scores for the results from the second implementation on the selected works. Table 7.1 gives two separate scores to show the effects of consolidation and cadence detection on the scores individually — note that this separation is not necessary for the Bach chorales since all cadences are already detected for the chorales. All pieces showed significant improvement with the addition of the consolidation process, with scores nearing or arriving at 0. Haydn’s *Tantum Ergo Sacramentum* showed little change in between consolidation and cadence detection primarily due to the prevalence of full rests in both works (as captured by a change described in section III), but Haydn’s *Deutschlandlied* and Naujalis’s *Vexilla Regis Prodeunt* both showed improvement with the addition of the cadence detection process.

As a means of validating the robustness of the algorithm, I introduced four additional scores with which to test both implementations, as shown in Table 7.3: one additional Bach chorale, a Mozart chorus, and two harmonizations of hymn tunes found in any standard hymnal.

The Mozart and the *Leoni* hymn yielded the addition of another consolidation rule due to all voices starting on the same note (F and C, respectively), which can safely be interpreted as a V to the subsequent full chord (B-flat and Fm, respectively). The analysis of the *Easter* hymn did not quite achieve perfection, showing a case of the tonal center detection being slightly over-sensitive: the chord after the first ‘alleluia’ is interpreted as a tonal center of F based on the touched cadence successor rule, but it should be interpreted as a continuation of the tonal center of C (see Fig. 7.3). An appendix has been included that contains all the results of both implementations of the algorithm on all the pieces referenced for the reader’s perusal.

Table 7.3: Additional scores tested after both implementations written

Work	Score on first implementation	Score on second implementation
BWV 248	-53	0
Mozart - <i>Abendruhe</i>	-6	0
<i>Easter</i>	-18	-3
<i>Leoni</i>	-18	0

The figure shows a musical score excerpt from *Easter* in 2/4 time. It consists of two staves: a treble staff and a bass staff. A purple box highlights a region in the first system, and a red box highlights a region in the second system. Below the score is a chord analysis table for keys C, F, and G.

C:	I	-	-	-	-	ii ⁷	vii ^{o6}	I ⁶
F:	-	I ⁶	V	I	V ⁶	vi ⁷	-	-
G:	-	-	-	-	-	-	-	-

Figure 7.3: A (cross-staff) excerpt from *Easter* showing a slight over-sensitivity in the tonal center detection process: the region interpreted in F should be in C (region in red; false center in purple); to see approaching phrase, refer to the appendix

VIII. FUTURE WORK AND APPLICATIONS

This project suffered from two primary limitations in the implementation of Taube's algorithm. First, this project could have benefited greatly from the existence of a dedicated NodeJS library for the parsing of Lilypond files. Given the absence of such a library and the limited time in completing this project, the score parsing comes with some restrictions, including the need for comments to be stripped from the file before parsing and the inability to correctly parse files using German note names (where 'h' represents B and 'b' represents B-flat). Second, the limited selection of common practice choral works on Mutopia restricted the number of pieces on which the implementations were tested and may have caused some consolidation rules needed for other common practice works to go unnoticed and therefore unimplemented. With a broader selection of works, the consolidation process could have become much more robust; as such, it remains unclear how well the process applies to common practice music in general.

As briefly mentioned in section III, the main motivation behind the selection of NodeJS for the algorithm implementation was to implement the algorithm with a Web-friendly language in order to prepare it for the inevitable Web applications that could use the algorithm. With a NodeJS implementation, for example, an ExpressJS server-side API could call upon the algorithm and return the algorithm's results to the client calling the API. Some future work in regard to better Web-friendliness includes transitioning from Lilypond engraving to MusicXML, a music engraving language built for the Web, which would allow for analysis results to be included in a webpage.



Figure 8.1: an example of a harmony writing exercise

Allowing for the algorithm on the Web will open the doors to online-based music theory exercises which include immediate feedback, much the same way Khan Academy math exercises provide immediate feedback to the student. Two exercise possibilities come to mind, both of which are currently performed almost exclusively in the traditional theory classroom. Figure 8.1 illustrates an example of a harmony writing exercise, where students are provided with a melody and a figured bass line which indicates the expected harmonic sequence for the phrase. In these exercises, students are tasked with filling out a four-part harmony within the melody and bass to match that expected harmonic sequence. With a Web-based exercise, a student's answer could be sent to the analysis algorithm and its output compared with the expected analysis, then the program could notify the student of any discrepancies in their solution's harmony. For example, if a student forgot to include an F in the penultimate chord for Fig. 8.1, the program could respond by telling the student the penultimate chord is missing an expected seventh, at which point the student could immediately correct this error and submit again.



Figure 8.2: an example musical phrase slated for analysis

Figure 8.2 provides an example problem for a second type of exercise: direct analysis of simple musical phrases. In this exercise, the student is presented with a short musical phrase to analyse by specifying the key and providing the Roman numeral analysis for the phrase. The example phrase provided in Fig. 8.2 was taken directly from a hymnal, but this exercise could theoretically involve the automatic creation of a simple musical phrase, much the same way Khan Academy exercises include randomly generated numbers for variables to ensure no two questions are the same. With such a phrase-creation scheme, the analysis algorithm could be called on the generated musical phrase once, then its output would be compared with the analysis provided by the student. . As with the harmony-writing exercise, the comparison would guide how the program responds to the student — for example, if the student marked the Dm7 chord on the third beat of the first measure in Fig. 8.2 as ‘ IV ’, the program would respond by indicating that figure to be incorrect, at which point the student could then immediately change the offending figure and resubmit their answer.

The updated implementation of Taube’s algorithm outlined in this paper has opened the door to higher-level analysis of homophonic choral music that can be leveraged as part of an immediate-feedback-loop system of music theory education. The implementation still has a few shortcomings to be addressed in the short-term, but once they are addressed the implementation has clear potential applications. Once applied on a broad basis, the implementation can grow more robust and become better equipped for all of common practice choral music, all the while helping music theory students learn their craft more efficiently.

REFERENCES

- [1] H. Taube, "Automatic Tonal Analysis: Toward the Implementation of a Music Theory Workbench," *Computer Music Journal*, vol. 23, no. 4, pp. 18–32, 1999.

APPENDIX

The following pages contain images of the scores analysed, ordered with the results of the first implementation and the results of the second implementation next to each other. The pieces included are, in order:

- Bach: BWV 248 (*Brich an, o schönes Morgenlicht*)
- Bach: BWV 259 (*Ach, was soll ich Sünder machen*)
- Bach: BWV 264 (*Als der gütige Gott vollenden wollt' sein Wort*)
- Bach: BWV 269 (*Aus meines Herzens Grunde*)
- Bach: BWV 277 (*Christ Lag In Todesbanden*)
- Bach: BWV 347 (*Ich dank dir, lieber Herre*)
- Haydn: *Deutschlandlied*
- Haydn: *Tantum Ergo Sacramentum*
- Mozart: *Abendruhe*
- Naujalis: *Vexilla Regis Prodeunt*
- *Easter hymn*
- *Leoni hymn*

Note that a few errors occurred on Lilypond's part in the typesetting of some of the second implementation results — they are not indicative of errors in the Lilypond files themselves, and I was unable to ascertain their cause. The errors are clarified below:

- BWV 259, m. 8: $ii^{\flat 6}_5$ on beat 1, V on beat 2
- BWV 277, m. 5: vi on beat 1, vi^7 on beat 2
- BWV 347, m. 2: i on beat 1, V^7 on beat 2

- BWV 347, m. 4: I_4^6 on beat 1, V^7 on beat 2
- *Deutschlandlied*, m. 11: V^7 on beat 3, V^7 on beat 4
- *Easter*, m. 7: I_4^6 on beat 1, V^7 on beat 2
- *Easter*, m. 15: I_4^6 on beat 1, V^7 on beat 2
- *Easter*, m. 17: I on beat 1, V_4^6 on beat 2
- *Easter*, m. 19: I^6 on beat 1, vii^{o6} on beat 2
- *Easter*, m. 23: I_4^6 on beat 1, V^7 on beat 2

The code base is included as a supplement to this paper on the NC Docks library archive.

To run the first basic implementation on a score, execute the following on the command line:

```
node analyse.js <lilypond_score> 0
```

To run the improved implementation, execute the following:

```
node analyse.js <lilypond_score> 1
```

D: - - - - - IV V₂⁴ I⁶ I₅⁷ V⁴ I₄⁶ V⁷ I - - - - -
 E: - - - - - V⁶ V₅⁶ VI⁶ ii^{ø7} III - - - - -
 G: I I₂⁴ vi IV⁶ V⁷ V I - - - - - I IV⁷ V₂⁴ iii⁷ I₂⁷ I V⁴ V V⁷ I
 A: - - - - -
 B: - - - - -

- - - - - iv vii^{ø7} I⁴ I⁴ I - - i V₃⁴ i⁶ VI₃⁴ - - - - - i₄⁶ V⁷ i - -
 IV i₄⁵ IV i⁶ ii⁷ vii^{ø6} i i - - - V V⁷ i - - - - - ii V₃⁴ - - - - - V V₂⁴
 - - - - - III₃⁴ V₅⁶ V⁶ i - - - - - vii^{ø7} V₅⁶ I⁴ iv₄⁶ - - - - -

- - - - - ii V₅⁶ IV⁶ V₅⁶ V⁷ I - - - - -
 I⁶ V₄⁷ I I₂⁴ IV₅⁶ vii^ø I vi - - - - - I⁶ iii IV IV V⁷ iii⁶ IV₅⁶ vii^ø I I⁴ ii₅⁶ vii₃^{ø4} ii⁶ V I
 - - - - -
 - - - - -

D: - - - IV V₂⁴ I⁶ V⁴ V⁷ I - - - - -
E: - - - - - - - - - V₅⁶ VI⁶ ii⁷ III - - - - -
G: I⁷ IV₅⁶ V⁷ I - - - - - I IV⁷ V₂⁴ I₂⁴ V⁴ V⁷ I
A: - - - - - - - - - - - - - - - - - -
B: - - - - - - - - - - - - - - - - - -

- - - iv vii⁷ I⁴ - - - i V₃⁴ VI₃⁴ - - - V⁴ V⁷ i -
 IV IV vii⁶ i - - V⁴ V⁷ i - - III₃⁴ V₅⁶ ii V₃⁴ - - - V⁷
 - - - - - - - - - - - - - - - - - V₅⁶ I⁴ - - -

- - - ii⁶ V₅⁶ IV⁶ V₅⁶ I - - - - -
 I⁶ I⁷ IV₅⁶ vii⁶ vi⁶ - - - I₅⁶ IV V⁷ IV₅⁶ vii⁶ I ii₅⁶ V I
 - - - - - - - - - - - - - - - - - -

D:
 E:
 G: vi vi₂⁷ vi⁶ vi₂⁷ vi vi₂⁴ IV⁷ vi₄⁶ ii ii₂⁴ V⁶ V I I I I₂¹ I⁶ vi⁷ ii⁷ V⁷ I I⁶ ii₅⁶ ii⁷ V V V⁷ I
 B:

5

VII⁶ VII₂⁷ iv₂⁵ iv⁶ V⁷ V⁶ V⁶ i VI - ii₅⁶ ii⁷ V V V⁷ i
 iii
 iv iv₂⁷ iv⁶ iv₅⁷ i i₂⁷ i⁶ i ii₅⁶ ii⁷ V⁴ V i

9

.....
 iii V₃⁴ I I₂⁴ IV⁶ V₅⁶ I vi ii₅⁶ ii⁷ V V V⁷ I

 V₂⁴ I⁶
 - VII⁶ ii₂⁴ V⁶ i VI ii₅⁶ ii⁷ V V V⁷ I

D: - - - - -
E: - - - - -
G: vi vi⁶ vi⁷ IV⁷ ii⁷ V⁶ I I I I⁶ vi⁷ ii⁷ V⁷ I ii⁶₅ V⁷ I
B: - - - - -

5

VII⁶ iv⁷₅ V⁷ i VI ii⁶₅ V⁷ i - - - - -
 - - - - - iv iv⁶ i i⁶ iV⁶₅ i

9

- - - - - V⁴₂ I⁶ - - - - -
 - - - - - VII⁶ ii⁶₂ V⁶ i VI ii⁶₅ V⁷ I
 iii V⁴₃ I⁷ IV⁶ V⁶₅ I vi ii⁶₅ V⁷ I - - - - -
 - - - - - - - - - - - - - - -

D: - - - - -
 E: - - - - -
 G: I vi vi V⁴ V I I⁷ vi⁶ V V⁷ I V I IV ii⁶ iii⁶ I₄⁶ V⁴ V V⁷

4

I I I⁶ I ii⁷ vii^{o6} I⁶ I₅⁷ V⁴ VII V⁶ i i VI⁶ IV⁶ V₅⁶ I I V⁴ V V⁷

8

ii V₃⁴ ii vii^{o6} vii^{o6} I - - - - -
 - - - - - V I IV⁶ IV I₄⁶ V⁴ V V⁷ I

D: - - - - -
 E: - - - - -
 G: I vi V I V⁷ I V I ii⁶ I₄⁶ V⁷

4

I I I⁶ I vii^{o6} I⁶ V V⁶ i VI⁶ IV⁶ V₅⁶ I V⁷

8

- ii vii^{o6} I - - -
 vi vi - - V I IV⁶ I₄⁶ V⁷ I

C: - - - - -
 G: I I IV⁶viV⁶ I iii⁶V vi IVI⁷₅IV⁴₂vii^{o6}I V I V⁶vi⁶₅vii^{o6}I⁶ii⁶₅VV⁷I

8

I I I⁷ii vii^{o6}I⁶I⁶ I⁶V³₂I⁶V⁴₃I V vi vi iii⁶₅I⁶ii⁶ iii⁷I⁶I⁵₂I⁷₄V⁷I I⁷₂I⁶ - - - V⁶V⁷I

16

V⁶V⁶₅I I vii^{o6}I⁶I VV⁷vi vi⁴₂IV vii^{o7}₅I⁷₂I V I V⁵₂V⁶IV⁶vii^{o4}₂vi vi⁴₂ii⁶₅VV⁷I

C: - - - - -
 G: I I IV₅⁶ V⁶ I iii₅⁶ vi IV IV₂⁴ vii^{o6} I V I V⁶ vi₅⁶ vii^{o6} I⁶ ii₅⁶ V⁷

I I I⁷ vii₅⁶ I⁶ I⁶ I⁶ V₃⁴ I V vi vi iii₅⁶ ii⁶ ii⁶ V⁷ I I⁶ V⁶ V⁷

- I V₅⁶ I vii^{o6} I⁶ I V⁷ vi IV vii^{o7}₅ I₂⁷ V I V₂⁵ V⁶ vii^{o4}₂ vi⁷ ii₅⁶ V⁷ I

C: - - - - -
 D: - - - - - V⁶ i i⁶ vii^{o6}₅ iv⁶₄ V⁶ - V⁵₄ i V i⁶ V⁷₄ V i
 F: - - - - -
 G: - - - - -
 A: iv iv⁷ vii^{o4}₃ V⁴₂ i⁶ vii^{o6} i i ii^{o4}₂ V⁶₅ i VI ii^{o6}₅ V V⁷ I - - - - -

- - - - - vii^{o6}
 - - - - - VI⁶ vii^{o6}₅ V⁴₃ I⁶ - - - - -
 vi iii⁵₄ vi vi⁴₂ ii⁶ iii⁷ ii V⁴₃ vi I⁶₄ ii⁶₅ V I I iii⁴₃ vi IV⁶ - - - - -
 - - - - - V⁶ V⁶₅ - - - - -
 - - - - - V⁶ V⁶₅ i VI ii^{o6}₅ V i - - - - -

I⁴ I IV I⁷₃ IV⁶ VII I - - - - -
 - - - - - ii^{o4}₃ V i⁶ ii^{o6}₅ V V⁷ VI - - - - - III⁶ i⁶₄ V⁴₃ V⁷ I⁴ I V⁵₂ I
 - - - - - V vii^{o4}₃ - - - - - ii vii^{o6} I⁶ - - - - -
 - - - - - V⁶ V⁶₅ i - - - - -

C: - - - - -
D: - - - - - V⁶ i vii^{o6}₅ iv⁶₄ V⁶ i V i⁶ V⁷ i
F: - - - - -
G: - - - - -
A: iv⁷ V⁴₂ i⁶ vii^{o6} i ii^{o4}₂ V⁶₅ i VI ii^{o6}₅ V⁷ I - - - - -

- - - - - vii^{o6}
 - - - - - VI⁶₅ V⁴₃ - - - - -
 - - - - - vii⁷ ii⁶ iii⁷ ii V⁴₃ vi I⁴₄ ii⁶₅ V I I iii⁴₃ IV⁶₅ - - - - -
 - - - - - V⁶₅ - - - - -
 - - - - - V⁶₅ i VI ii^{o6}₅ V i - - - - -

I IV I⁷₃ IV⁶ VII I - - - - -
 - - - - - ii^{o4}₃ V i⁶ ii^{o6}₅ V⁷ VI - - - - - ii^{o6}₅ i⁶₄ V⁷ I
 - - - - - V vii^{o4}₃ - - - - -
 - - - - - V⁶₅ - - - - -
 - - - - - - - - - -

D: - - - - -
 E: IV IV₂⁴ ii IV⁶ IV⁶ VII VII V⁶ i V V⁷ I I IV iii⁶ V IV⁶ vi₂⁴ ii₅⁶ I₄⁶ V V⁷ I
 A: - - - - -
 B: - - - - -

5

- - - - - V⁶ V⁷ I - - - - -
 iii iii⁷ IV⁶ vii^o I IV₃⁴ V I⁶ - IV vii^{o6}₄ I⁶ I₅⁷ V - - - - - iii iii
 - - - - - vii^{o6} i i⁷ ii^{o7} vii^{o6}₅ i⁶ iv⁶ V i - - - - -

10

- - - - - vi V⁷ ii⁶ iii iii ii vii^{o6} I V₃⁷ I⁶ IV⁷ V ii vi iii₄⁶ ii₂⁴ vi⁶ V⁶ - vi₅⁶ IV₄⁷ I V⁶ vi⁷ ii₃⁴ I₄⁶ IV⁷ V V V⁷ I
 - - - - -

D: - - - - -
 E: IV⁷ ii IV⁶ VII V⁶ V⁷ I I IV iii₅⁶ IV⁶ vi₂⁴ ii₅⁶ IV⁷ I
 A: - - - - -
 B: - - - - -

5

- - - - - V₅⁶ I - - - - -
 iii⁷ IV⁶ vii^o IV₃⁴ V - IV vii₄^{o6} I⁶ V - - - - - iii
 - - - - - vii^{o6} i⁷ vii₅^{o6} i⁶ iv⁶ V i

10

- - - - -
 vi V⁷ iii vii₅^{o6} I V₃⁷ I⁶ IV⁷ V ii vi iii₄⁶ ii₂⁴ V⁶ vi₅⁶ vii₂^{o7} I vi⁷ ii₃⁴ I₄⁶ IV⁷ V⁷ I
 - - - - -

Langsam

mf

1. Deutsch-land, Deutsch-land ü - ber al - les, ü - ber al - les in der
 2. Deut - sche Frau - en, deut-sche Treu-e, deut-scher Wein und deut-scher
 3. Ei - nig - keit und Recht und Frei-heit für das deut-sche Va - ter -

D: - - - - - vii^{°7} vii[°]
 G: I V I V V₅⁶ I V⁷ V⁷ I IV I⁶ V₃⁴ V₅⁶ I ii⁶ - -

mf

Welt, wenn es stets zu Schutz und Trut-ze brü - der - lich zu - sam - men -
 Sang sol - len in der Welt be - hal - ten ih - ren al - ten schö - nen
 land! Da - nach lasst uns al - le stre - ben brü - der - lich mit Herz und

I IV I IV I - - - - - vii^{°7} vii[°]
 - - - - - V V₅⁶ I V⁷ V⁷ I IV I⁶ V₃⁴ V₅⁶ I ii⁶ - -

p *cresc.* *f*

hält, von der Maas bis an die Me-mel, von der Etsch bis an den Belt,
 Klang, uns zu ed - ler Tat be - geist-ern un - ser gan - zes Le - ben lang.
 Hand! Ei - nig - keit und Recht und Frei-heit sind des Glü - ckes Un - ter - pfand.

I I IV I I I - - - - - ii V⁷ V⁷ I⁶ I
 - - - - - V V₅⁶ I V V V iii ii⁶ V⁷ vi vi vi vi - - - -

ff

Deutschland, Deutschland ü - ber al - les, ü - ber al - les in der Welt.
 Deut - sche Frau - en, deutsche Treu-e, deut - scher Wein und deut-scher Sang.
 Blüh im Glan - ze die - ses Glückes, blü - he, deut-sches Va - ter - land!

IV⁶ IV₅⁶ - - - - -
 - I₅⁶ IV⁴ IV I IV I V⁶ V₅⁶ I V V₂⁴ I₅⁷ V₂⁴ I⁶ vi₃⁴ IV ii⁶ I₄⁶ V⁷ I₃⁷ I

Langsam

mf

1. Deutsch-land, Deutsch-land ü - ber al - les, ü - ber al - les in der
 2. Deut - sche Frau - en, deut-sche Treu-e, deut-scher Wein und deut-scher
 3. Ei - nig - keit und Recht und Frei-heit für das deut-sche Va - ter -

D: - - - - - vii^{o7}
 G: I V I V V₅⁶ I V⁷ I IV I⁶ V₃⁴ I ii⁶ -

mf

Welt, wenn es stets zu Schutz und Trut-ze Brü-der - lich zu - sam - men -
 Sang sol - len in der Welt be - hal - ten ih - ren al - ten schö - nen
 land! Da - nach lasst uns al - le stre - ben Brü-der - lich mit Herz und

I - I V I V V₅⁶ I V⁷ I IV I⁶ V₃⁴ I ii⁶ -

p *cresc.* *f*

hält, von der Maas bis an die Me-mel, von der Etsch bis an den Belt,
 Klang, uns zu ed - ler Tat be - geist-ern un - ser gan - zes Le - ben lang.
 Hand! Ei - nig - keit und Recht und Frei-heit sind des Glü - ckes Un - ter - pfand.

I - V I V V V₅⁶ I V V iii ii⁶ V⁷ vi vi - I

ff

Deutschland, Deutschland ü - ber al - les, ü - ber al - les in der Welt.
 Deut - sche Frau - en, deutsche Treu-e, deut - scher Wein und deut - scher Sang.
 Blüh im Glan - ze die - ses Glückes, blü - he, deut - sches Va - ter - land!

I⁶ I₅⁶ IV I IV I V₅⁶ I V V₂⁴ vi₃⁴ ii₅⁶ I₄⁶ V⁷ I

Andante

mf
 Tan-tum er - go sa - cra - men-tum ve - ne - re - mur cer - nu - i
 Ge - ni - to - ri ge - ni - to - que la - us et ju - bi - la - ti - o

C: I I I IV⁶₄ IV⁶₄ I⁷ vi⁶ I⁷ IV⁶₄ IV⁶₄ I I I V⁶₄ I⁶ IV IV ii⁶ I⁴₄ V I
 G: -

9

et an-ti-quum do - cu - men-tum no - vo ce - dat ri - tu - i. Prae-stet
 sa - lus, ho - nor, vir - tus quo - que sit et be - ne - di - cti - o. Pro - ce -

IV IV⁶ IV IV⁶₄ I I vii⁷ ii⁷ vii⁷ ii⁴₂ vi⁶ I I I IV I⁶ ii⁶ I⁴₄ V⁷ I - -
 I I

18

fi - des sup - ple - men-tum sen - su - um de - fec - tu - i.
 den - ti ab u - tro - que com - par sit lau - da - ti - o. A - men

I IV⁶₄ IV⁶₄ I⁷ vi⁶ I⁷ IV⁶₄ IV⁶₄ I I I V⁶₄ I⁶ IV IV ii⁶ I⁴₄ V I IV I

Andante

mf

Tan-tum er-go sa-cra-men-tum ve-ne-re-mur cer-nu-i
 Ge-ni-to-ri ge-ni-to-que la-us et ju-bi-la-ti-o

C: I I IV⁶₄ IV⁶₄ I⁷ vi⁶ I⁷ IV⁶₄ I I I V⁶₄ I⁶ IV IV ii⁶ I⁶₄ V I
 G: -

9

pp

et an-tiqu-um do-cu-men-tum no-vo ce-dat ri-tu-i. Prae-stet
 sa-lus, ho-nor, vir-tus quo-que sit et be-ne-di-cti-o. Pro-ce-

IV IV⁶ IV I I vii^{o7} ii⁷₅ vii^{o7} vi⁶ I I I IV I⁶ ii⁶ I⁶₄ V⁷ I - -

18

f *pp*

fi-des sup-ple-men-tum sen-su-um de-fec-tu-i.
 den-ti ab u-tro-que com-par sit lau-da-ti-o. A-men

IV⁶₄ IV⁶₄ I⁷ vi⁶ I⁷ IV⁶₄ I I I V⁶₄ I⁶ IV IV ii⁶ I⁶₄ V I IV I

1. Ver-klun- gen ist des Ta- ges Trei- - ben, nicht lang mehr will die
 2. Viel Ster- ne klar am Him- mel schim- - mern, viel Her- zen bang auf
 3. Er, der am A- bend al- les dek- - ket, ob Leid, ob Freud der

Eb: - - - - -
 F: - - - - -
 G: - - - - -
 Bb: - I I V₃⁴ I V⁶ vii^o V₅⁶ I IV₄⁶ I I V₃⁴ I V⁶ I

Son- ne blei- - ben, von ih- rer Ar- beit ruht die Hand;
 Erd' sich küm- - mern um Er- den- leid und Er- den- weh:
 Tag ge- wek- - ket, in Feld und Wald mit Schat- ten zu,

I₄⁶ V⁷ V⁷ vii^{o7}₂ I I - - - - -
 V₄⁶ - - - - - V V V I₃⁴ vi₂⁴ vi₂⁴ V⁷ V⁷ V⁷ iii₅⁶ I₄⁶

13 der Fei- er- a- bend deckt das Land, der Fei- er- a- bend deckt das Land.
 mein Herz, blick' auf zur Him- mels- höh'; mein Herz, blick' auf zur Him- mels- höh'!
 er- füllt auch dich mit Fried' und Ruh', er- füllt auch dich mit Fried' und Ruh'.

- - - - - I₄⁶ V⁷ I - - - - - *pp*
 V₅⁶ V₅⁶ V₅⁶ i iv⁶ III₄⁶ - - - - -
 - - - - - I₄⁶ V⁷ I I IV₄⁶ - IV IV I₄⁶ V⁴ V⁷ I

1. Ver-klun- gen ist des Ta- ges Trei- - ben, nicht lang mehr will die
 2. Viel Ster- ne klar am Him- mel schim- - mern, viel Her- zen bang auf
 3. Er, der am A- bend al- les dek- - ket, ob Leid, ob Freud der

E \flat : - - - - -
 F: - - - - -
 G: - - - - -
 B \flat : V I I V $\frac{4}{3}$ I V 6 V 6_5 I I I V $\frac{4}{3}$ I V 6 I

Son- ne blei- - ben, von ih- rer Ar- beit ruht die Hand;
 Erd' sich küm- - mern um Er- den- leid und Er- den- weh:
 Tag ge- wek- - ket, in Feld und Wald mit Schat- ten zu,

I 6_4 V 7 V 7 I I - - - - -
 V 6_4 - - - - - V V V vi $\frac{4}{2}$ vi $\frac{4}{2}$ V 7 V 7 V 7 iii 6_5 I 6_4

der Fei- er- a- bend deckt das Land, der Fei- er- a- bend deckt das Land.
 mein Herz, blick' auf zur Him- mels- höh'; mein Herz, blick' auf zur Him- mels- höh'!
 er- füllt auch dich mit Fried' und Ruh', er- füllt auch dich mit Fried' und Ruh'.

- - - - - V I 6_4 V 7 I - - - - -
 V 6_5 V 6_5 V 6_5 i iv 6 III 6_4 - - - - -
 - - - - - I 6_4 V 7 I - - - - - IV IV I 6_4 V 4 V 7 I

1. Vex - il - la re - gis pro - de - unt, ful - get cru - cis my -
 2. Quo vul - ne - ra - tus in - su - per quæ vul - ne - ra - ta
 3. Im - ple - ta sunt, quæ con - ci - nit Da - vid fi - de - li
 4. Ar - bor de - co - ra et ful - gi - da, or - na - ta Re - gis
 5. Be - a - ta, cu - jus bra - chi - is pre - tium pe - pen - dit
 6. O crux a - ve, spes u - ni - ca, hoc Pas - si - o - nis
 7. Te fons sa - lu - tis Tri - ni - tas, col - lau - det o - mnis

D: - - - - - ii⁷ V₃ vii^{o6} I I - - - - -
 E: -
 G: I vi vi I⁶ iii ii⁶ vi vi⁷ - - - - - V ii⁶ I⁶ ii vii^{o6} I

1. ste - ri - um, qua vi - ta mor - tem per - tu - lit,
 2. lan - ce - æ mu - cro - ne di - ro cri - mi - num,
 3. car - mi - ne, di - cen - do na - ti - o - ni - bus:
 4. pur - pu - ra, e - lec - ta di - gno sti - pi - te
 5. sæ - cu - li, sta - te - ra fa - cta cor - po - ris,
 6. tem - po - re: pi - is a - dau - ge gra - ti - am,
 7. spi - ri - tus: qui - bus cru - cis vi - cto - ri - am

- - - - - - - - - - - V₅⁶ I
 - iv V i i i VI⁶ III⁶ - - - - -
 vii^{o6} ii - - - - - I⁶ iii vi IV IV⁷ - V

1. et mor - te vi - tam pro - tu - lit.
 2. ut nos la - va - ret cri - mi - ne.
 3. re - gna - vit a li - gno De - us.
 4. tam san - cta mem - bra tan - ge - re.
 5. tu - lit - que præ - dam tar - ta - ri.
 6. re - is - que de - le cri - mi - na.
 7. lar - gi - ris, ad - de præ - mi - um. A - men.

I vi V⁶ I iii ii⁶ vi ii₅⁶ V I IV I

1. Vex - il - la re - gis pro - de - unt, ful - get cru - cis my -
 2. Quo vul - ne - ra - tus in - su - per quæ vul - ne - ra - ta
 3. Im - ple - ta sunt, quæ con - ci - nit Da - vid fi - de - li
 4. Ar - bor de - co - ra et ful - gi - da, or - na - ta Re - gis
 5. Be - a - ta, cu - jus bra - chi - is pre - tium pe - pen - dit
 6. O crux a - ve, spes u - ni - ca, hoc Pas - si - o - nis
 7. Te fons sa - lu - tis Tri - ni - tas, col - lau - det o - mnis

D: - - - - - ii⁷ V₃⁴ I - - - - -
 E: -
 G: I vi vi I⁶ iii ii⁶ vi vi⁷ - - - - - V ii⁶ I⁶ ii vii^{o6} I

1. ste - ri - um, qua vi - ta mor - tem per - tu - lit,
 2. lan - ce - æ mu - cro - ne di - ro cri - mi - num,
 3. car - mi - ne, di - cen - do na - ti - o - ni - bus:
 4. pur - pu - ra, e - lec - ta di - gno sti - pi - te
 5. sæ - cu - li, sta - te - ra fa - cta cor - po - ris,
 6. tem - po - re: pi - is a - dau - ge gra - ti - am,
 7. spi - ri - tus: qui - bus cru - cis vi - cto - ri - am

- - - - - V₅⁶ I
 - - - - - iv V i - - - - - - - - - - - - - - -
 vii^{o6} ii - - - - - vi vi IV⁶ I⁶ iii vi IV IV⁷ - - - - -

1. et mor - te vi - tam pro - tu - lit.
 2. ut nos la - va - ret cri - mi - ne.
 3. re - gna - vit a li - gno De - us.
 4. tam san - cta mem - bra tan - ge - re.
 5. tu - lit - que præ - dam tar - ta - ri.
 6. re - is - que de - le cri - mi - na.
 7. lar - gi - ris, ad - de præ - mi - um. A - men.

I vi V⁶ I iii ii⁶ vi ii₅⁶ V I IV I

Musical notation system 1 (measures 1-10) with chord symbols below:

C: I I V⁶ I IV⁶ IV IV₄⁶ I I V₃⁷ I⁶ I⁶ V₃⁷ V₃⁴ I IV⁶ I₄⁶ V V⁷ I IV⁶ I IV I⁶

G: -

Musical notation system 2 (measures 11-21) with chord symbols below:

ii⁷ vii^{o6} I⁶ I₅⁶ I₄⁶ V I V₃⁷ I⁶ I⁶ V₃⁷ V₃⁴ I IV⁶ I₄⁶ V V⁷ I V ii₃⁷ - - - V V₂⁴ I⁶ ii⁷ vii^{o6} I V ii₃⁷ V⁶ V⁶

- -

Musical notation system 3 (measures 22-31) with chord symbols below:

ii₃⁷ -

V₃⁷ V₃⁴ I IV⁶ I₄⁶ V V⁷ I - vi₂⁴ V₂⁴ V₂⁴ I⁶ I I₂⁴ IV⁶ IV IV₄⁶ I I V₃⁴ I⁶ I IV V⁷ vi ii₅⁶ I₄⁶ V V⁷ I

I ii₂⁴ -

C: I I V⁶ I IV⁶ IV IV₄⁶ I I V₃⁷I⁶ V₃⁴ IIV⁶ V₄⁷ I - - - -
 F: - I⁶ V I V⁶
 G: -

11

ii⁷ vii^{o6} I⁶ I₄⁶ V I V₃⁷I⁶ V₃⁴ IIV⁶ V₄⁷ I - V⁶ V⁷ vii^{o6} I - - - -
 vi⁷ -
 - V₃⁴ I⁶ - - - - I V₃⁷I⁶

22

- - - - - V V₂⁴ I⁶ I⁷ IV⁶ IV IV₄⁶ I I V₃⁴I⁶ IV V⁷ vi ii₅⁶ I₄⁶ V⁷ I
 -
 V₃⁴ IIV⁶ V₄⁷ I -

F: - i V i iv⁶ V III VII⁶ III VI⁶ VII III VII III⁶ VII III VI⁶
 Ab: -

6

VII - - - - - VII V V⁷
 V V₂⁴ I⁶ IV⁶ I₄⁶ IV⁷ V V⁷ I IV ii₅⁶ I I₄⁶ I⁶ I V - -

11

i i VI iv⁶ i⁶ iv V⁴ V V i⁶ V i iv⁶ i⁶ iv⁶ i₄⁶ iv i⁶ iv i₄⁶ iv⁷ V⁴ V i
 -

F: V i V i iv⁶ V - - - - - I V⁶ I IV⁶ V I V I⁶ V I IV⁶ V V⁴/₂ I⁶
 Ab: - - - - -

- - - - - iv⁶/₅ III III⁶/₄ III⁶ III VII V⁷ i iv⁶/₅ i⁶ iv
 IV⁶ I⁶/₄ IV⁷ V V⁷ I - - - - -

12

V⁴ V V i⁶ V i iv⁶ i⁶ iv⁶ i⁶/₄ iv i⁶ iv i⁶/₄ iv⁷ V⁴ V i
 - - - - -